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OPTIMIZED ELECTRIC POWER SUBSTATION LOCATION USING P-MEDIANS AND VORONOI DIAGRAMS

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Abstract. *To face the growing demand for electric energy in large metropolitan areas, it is necessary the construction of new power substations and the extension of those already in operation. Electric power companies develop a planning to define the quantity, installed power and location of power substations necessary to meet current and future demand of electric distribution network within a region. Long term planning is a complex task that includes not only technical issues but also geographical, political and economical issues and has to be updated at least yearly. This planning requests the identification of loads in the area and current capacity and expansion capability of substations. Loads are allocated to a given substation according to the distance to it and other technical criteria. The main objective is to meet demand with the minimum transportation cost, defined by the sum of the product of the loads by its distance to the serving substation. This cost is optimized for all substations in the area, basically by allocating loads to substations. Prior to this allocation, it is necessary to locate substations in such way that future allocations of load will yield the minimum cost. Therefore, planning the expansion of the electric distribution network is a rather complex real-world optimization problem. This work aims at developing a method using p-medians and Voronoi diagrams to aid engineers finding optimized solutions. An optimized plan will reduce investments, meet demand and keep satisfactory quality of services. The proposed method was tested with real data from the central area of Curitiba, Brazil, comprising 42 substations. Results of the method were compared with the State-owned electric company studies, as well as other methodology. The proposed method is competitive regarding accuracy, and computationally efficient. The method is promising, allowing a significant reduction of planning efforts by technical staff, besides offering feasible optimized solutions.*

Keywords: *Optimization, p-medians, Voronoi diagrams, Electric power distribution*

1. INTRODUCTION

To face the growing demand for electric energy in large metropolitan areas, it is necessary the construction of new power substations and the extension of those already in operation.

Electric power distribution companies develop a planning to define the quantity, installed power and location of power substations necessary to meet current and future demand of the electric distribution network within a region. Long term planning is a complex task that includes not only technical issues but also geographical, political and economical issues. Due to the dynamic nature of the facts the planning is based on, it is timely updated, usually once a year. To do the plan, it is necessary to identify and quantify loads in the area to be served by a power substation, considering its current capacity and expansion capability. Loads are allocated to a given substation according to the distance to it, its available capacity and technical characteristics of the electric distribution network. The main objective is to meet demand with the minimum transportation cost, defined by the sum of the product of the loads by its distance to the substation. This cost must be optimized for all substations in the area, basically by allocating loads to substations. Prior to this allocation, it is necessary to locate substations in such way that future allocations of load will yield the minimum cost. Planning the location of new substations must take into account the existing substations and the network, besides the distribution of loads, current and future. Therefore, planning the expansion of the electric distribution network is a rather complex real-world optimization problem.

This work aims at developing an algorithm to optimize the location of electric power substations so as to give alternatives to the technical staff in charge of planning the expansion of the network. By means of such algorithm, their work will be more efficient, enabling optimized solutions. As a consequence, costs and investments will be decreased leading to more profitability of the company.

The developed algorithm uses the Teitz and Bart's algorithm for the p-median problem (Teitz & Bart, 1968) to find suitable locations for substations, and the Voronoi diagram (Aurenhammer, 1988) to allocate loads to substations, according to their capacities. Therefore, it is aimed to satisfy demands with the smallest possible transportation cost.

2. ELECTRIC POWER DISTRIBUTION

The electrical power generated by a power plant (hydraulic, thermal, nuclear, etc) is transported to cities by means of transmission lines, usually with voltages above 69 kV (69,000 Volts). In the urban regions, next to the demand areas, such voltages are reduced to more appropriated levels for distribution to consumers – voltages around 13.8 and 34.5 kV. This is done in a power substation.

Any consumption of electric energy is considered a load for the electric network. In this work, to simplify the computation of load allocation, loads in a region are grouped into small square areas of 1 km^2. All loads inside the square are grouped in its center and considered as a single point. The summation of the product of point loads by its respective distance to the serving substation is known as "electric moment". This is the basic cost function used in this work, which is aimed at minimizing.

2.1 Related work

In a recent work, Correa (2003) compared the p-medians method (Teitz & Bart, 1968) and the c-means algorithm to find appropriate locations for substations. Also, he compared two algorithms for assigning loads to substations: Gillet & Johnson (Bodin, Golden, Assad et al, 1983) and Ford & Fulkerson (Ford & Fulkerson, 1962). The first conclusion was that p-medians are more suitable do find locations for substations although it takes more time to run. A second conclusion was that Ford & Fulkerson algorithm provides better results than Gillet & Johnson.

Miguez, Diaz-Dorado and Cidras (1998) proposed a solution for substation location optimization using evolutionary strategies. According to them, the investments in power distribution systems constitute a significant part of the utilities expenses. They can account up to 60 percent of capital budget and 20 percent of operating costs. For this reason, efficient planning tools are needed to allow planners to reduce costs. The method proposed is called *(m+l)-ES*, which is characterized by a population of m individuals from which two are randomly selected that give rise to two descendants, through the successive application of a crossover operator and a mutation operator, thus resulting in a larger population of $m+l$ individuals. This algorithm has been used to determine the set of substations needed to supply the whole Spain.

3. P-MEDIANS

The p-medians problem is widely studied in the recent literature and it aims at locating p facilities (medians) amongst a set of n ($n \geq p$) possible locations, in such a way that the sum of all distances from each demand point to its closest median is minimal (Kariv & Hakimi, 1979). This criterion is known as *minisum* and it is usually associated with facility location problems (Drezner & Hamacher, 2004).

Let $G(V,E)$ be a graph where $V=\{v_1, v_2, ..., v_n\}$ is the set of of vertices (nodes) and $E=\{e_{ij}\}$ is the set of edges. The number of *out*-transmission and *in*-transmission for each vertex $v_i \in V$ are defined by Eq. 1 and Eq. 2, respectively (Christofides, 1975):

$$\sigma_o(v_i) = \sum_{v_j \in V} \theta_j . d(v_i, v_j) \tag{1}$$

$$\sigma_t(v_i) = \sum_{v_j \in V} \theta_j . d(v_j, v_i) \tag{2}$$

where: $d(v_i, v_j)$ is the shortest distance between vertices v_i and v_j, and θ_j is the weight associated to vertex v_j. We call *out*-median and *in*-median of a graph, respectively, to vertices v_o and v_t that satisfy the conditions, given by Eq. 3 and Eq. 4:

$$\sigma_o(v_o) = \min_{v_i \in V}[\sigma_o(v_i)] \tag{3}$$

$$\sigma_t(v_t) = \min_{v_i \in V}[\sigma_t(v_i)] \tag{4}$$

In order to generalize the concepts of *out*-transmission and *in*-transmission to p-medians, let V_p to be a subset of set V with p elements, that is, $|V_p| = p$. We define:

$$d(V_p, v_j) = \min_{v_i \in \mathrm{V_p}}[d(v_i, v_j)] \tag{5}$$

$$d(v_j, V_p) = \min_{v_i \in \mathrm{V_p}}[d(v_j, v_i)] \tag{6}$$

where: $d(V_p, v_j)$ represents the distance from the subset of vertices V_p until vertex v_j, and $d(\mathrm{v_j}, V_p)$ the opposite.

Analogously to what was done for a single vertex, the number of *out*-transmission and *in*-transmission for the set V_p are defined, respectively, by Eq. 7 and Eq. 8, as follows:

$$\sigma_o(V_p) = \sum_{v_j \in V} \theta_j . d(V_p, v_j) \tag{7}$$

$$\sigma_t(V_p) = \sum_{v_j \in V} \theta_j . d(v_j, V_p) \tag{8}$$

Finnaly, it is called *p-out*-median and *p-in*-median, respectively, sets V_{po} and V_{pt}, for which holds Eq. 9 and Eq. 10:

$$\sigma_o(V_{po}) = \min_{V_p \subset V} [\sigma_o(V_p)] \tag{9}$$

$$\sigma_o(V_{pt}) = \min_{V_p \subset V} [\sigma_o(V_p)] \tag{10}$$

For undirected graphs, it does not matter to consider either set *p-out*-median or *p-in*-median, which can be called simply the set of *p*-medians.

3.1 The Teitz and Bart algorithm for the *p*-medians

First, a set *S* with *p* vertices is chosen, considering an approximation to the *p*-medians set V_p. Next we verify if any vertex $v_i \in V - S$ can substitute (according to the algorithm shown below) some vertex $v_j \in S$, producing a new set S' such that: $S' = S \cup \{v_i\} - v_j\}$ and $\sigma(S') < \sigma(S)$. In the case this is possible, v_j is substituted by v_i and S' is considered a new approximation to the set V_p. The algorithm proceeds until the actual set *S* is obtained, when no substitution of vertices can produce a smaller transmission number. The steps for the algorithm are:

Step 1. Build an initial set *S* with *p* elements of *V*;

Step 2. Label all vertices $v_i \notin S$ as “not-analyzed”;

Step 3. While there is “not-analyzed” vertices in the set *V-S*, do:

a) Select a “not-analyzed” vertex $v_i \in V\text{-}S$, and compute the reduction Δ_{ij} of the transmission number $\forall\ v_j \in S$: $\Delta_{ij} = \sigma(S) - \sigma(S \cup \{v_i\} - \{v_j\})$;

b) Do $\Delta_{ij_o} = \max_{v_j \in S} [\Delta_{ij}]$;

c) If $\Delta_{ij_o} > 0$ do $S \leftarrow S \cup \{v_i\} - \{v_{j_o}\}$, and label v_{j_o} as “analyzed”;

d) If $\Delta_{ij_o} \leq 0$ label v_i as “analyzed”.

Step 4. If, during the execution of the preceding step, there were modifications in set *S*, go back to step 2. Otherwise, stop. Set *S* will be the approximate *p*-medians set.

4. VORONOI DIAGRAMS

4.1 Common Voronoi diagram in the plane

Given a finite set of two or more distinct points in the continuous Euclidean plane $P = p_1, p_2, \ldots, p_n$, all regions of this space are associated to the closest element(s) of *P*, with respect to the Euclidean distance. The result is a tecelation in the plane (see Fig. 1), constituted by regions associated with members of set *P*. This tecelation is called common Voronoi diagram in the plane generated by the set of points, and the regions that constitute the Voronoi diagram are the common polygons of Voronoi (Aurenhammer, 1988, Okabe et al., 1992).

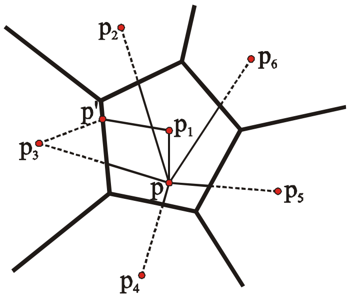

Figure 1 – Common Voronoi diagram in the plane.

A more formal definition of the Voronoi diagram is provided as follows. Let $P = \{p_1, p_2,..., p_n\}$, where $2 \leq n < \infty$ and $x_i \neq x_j$ for $i \neq j$, $i,j \in I_n$. The region defined by Eq. 1 is the common Voronoi polygon that is associated with p_i (or the p_i Voronoi polygon).

$$V(p_i) = \{x \,/\, |x - x_i| \leq |x - x_j|,\ for\ j \neq i,\ j \in I_n\} \tag{11}$$

The set defined by Eq. 2 is the Voronoi diagram (the common diagram in the plane), generated by P (the P Voronoi diagram):

$$V = \{V(p_1), V(p_2),..., V(p_n)\} \tag{12}$$

4.2 Multiplicative Voronoi diagram with weights

This type of Voronoi diagram is characterized by a weighted distance, given by the Eq. 13. This distance is known as a multiplicative weighted distance, or MW-distance (Galvão, 2003). The domain region with MW-distance is written as in Eq. 14.

$$d_{mw}(p,p_i) = \frac{1}{w_i}\|x - x_i\|,\ w_i > 0 \tag{13}$$

$$Dom(p_i,p_j) = \{x \,/\, \frac{1}{w_i}\|x - x_i\| \leq \frac{1}{w_j}\|x - x_j\|\},\ i \neq j \tag{14}$$

By using Eq. 14, we can obtain the mediatrix shown in Eq. 15:

$$b(p_i,p_j) = \{x \,/\, \|x - \frac{w_i^2}{w_i^2 - w_j^2}x_j + \frac{w_j^2}{w_i^2 - w_j^2}x_i\| = \frac{w_i w_j}{w_i^2 - w_j^2}\|x_j - x_i\|\},\ w_i \neq w_j,\ i \neq j. \tag{15}$$

Such mediatrix is the region around a point p such that it satisfies the condition that the distance from p to the fixed points, $w_i^2 x_j/(w_i^2 - w_j^2) - w_j^2 x_i/(w_i^2 - w_j^2)$, is constant. This region is defined by a circumference in $\Re^2$, over the points of interior and exterior division $\overline{p_i p_j}$ at the rate w_i, w_j. In the classical geometry, this circumference is known as the circle of Apollonius. Figure 2 shows the Apollonius circle representing the mediatrix defined with MW-distance for several rates $\alpha = w_i/w_j$ (it is assumed that $w_i/w_j \geq 1$, without losing generality). In the special case that $\alpha = 1$ ($w_i = w_j$), the mediatrix becomes a straight line (or a circle with infinite radius).

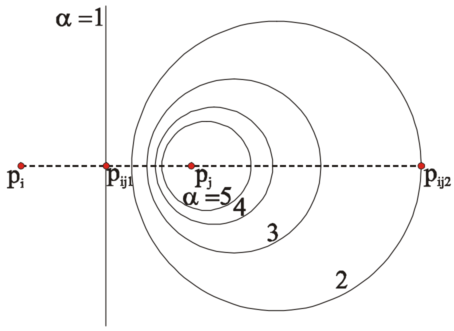

Figure 2 – Weighted multiplicative Voronoi diagram for 2 points.

Figure 3 shows an example with weighted multiplicative mediatrices for several $\alpha = w_i/w_j = 1, 2, 3, 4, 5$. This is also known as weighted multiplicative Voronoi diagram for $n = 2$. In this figure, numbers within parenthesis represent the weights associated to the generators.

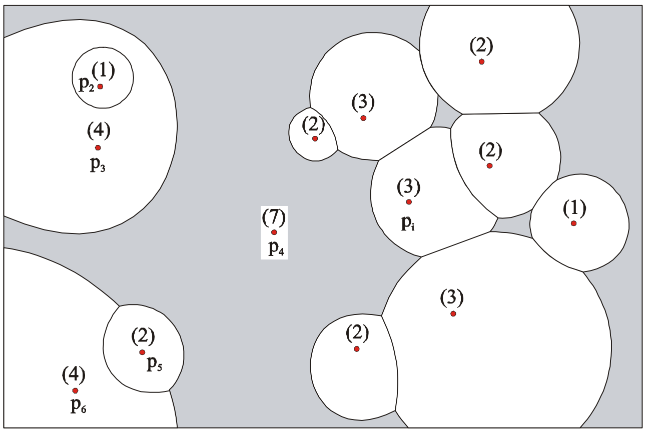

Figure 3 – Example of weighted multiplicative Voronoi diagram in $\Re^2$.

In Fig. 3, it can be observed the geometrical properties of a MW-Voronoi diagram. First, $V(p_1)$ is not convex. Second, $V(p_2)$ is encircled within $V(p_3)$, or $V(p_3)$ has a hole. Third, $V(p_4)$, represented by the shaded region, is not connected. Fourth, weights of the MW-Voronoi regions adjacent to a convex MW-Voronoi, such as $V(p_5)$, is not lower than the weight of the convex MW-Voronoi region ($w_5 < w_4,w_6$).

4.3 Power Voronoi diagram

The *Power* Voronoi diagram (Silva, 2004) is characterized by the weighted distance given by Eq. 16:

$$d_{mw}(p,p_i)= \|x-x_i\|+w_i,\ w_i>0 \tag{16}$$

This distance is known as weighted multiplicative distance or PV-distance. The domain region with PV-distance is written as:

$$\mathrm{Dom}(p_i,p_j)=\{x \,/\, (\|x-x_i\|+w_i)^{-1} \le (\|x-x_j\|+w_j)^{-1}\},\ i\neq j \tag{17}$$

5. METHODOLOGY

5.1 Problem characteristics

Prior to the development of a methodology to solve the problem, it is important to identify its main characteristics, regarding an optimization problem, as follows:

- The general objective is to find suitable location for several electric power substations;
- All substations are identical, from the point of view of their capability to serve loads;
- The problem is single-service. The only objective of substations (in this work) is to serve the network with electric power;
- This is a capacitated problem. That is, power substations have a limited capacity to serve loads. This constraint aggregates difficulty to the problem;
- The number of substations is predefined. This number is calculated by means of the division of the sum of all loads to be served in the region, by the capacity of a standard power substation;
- The location model is continuous. That is, substations can be located at any position within the region considered. Any possible restrictions regarding this issue (such as availability of area and geographical limitations) are not considered in this work. To simplify the problem, substations are always located at the center of the squares;
- The demand is distributed. That is, consumers (residential, industrial, etc) are grouped in squares of 1 km^2. These squares are evenly distributed over all the region under study;
- The demand is differentiated. That is, different squares can have different demands of electric power. Recall that this study covers a metropolitan area in which there are residential and industrial spots of different density;
- This is a single-demand problem. That is, consumers demand a single service – electric power.

5.2 Data used

Data used in this work were obtained from COPEL - Companhia Paranaense de Energia (http://www.copel.com), a State-owned electric power company for the State of Paraná, Brazil. The region under study comprises the metropolitan region of Curitiba. As mentioned before, the whole region was divided into squares. For each square, the company provided information about the demand of electric power. This information was obtained using current data of consumers and an estimative of future demand. In this work, the network of squares, previously defined by the company, was re-divided into smaller squares. That is, each original square (having 1000 x 1000 meters) was divided into l^2 smaller squares (having l x l meters), in such a way that the demand of these smaller squares will be a fraction (1/ l^2) of the original square.

5.3 Methods

We apply the power Voronoi algorithm, centered in n points, to a mesh representing the region under study so as to obtain the initial assignment of squares (representing loads) to substations. Therefore, n regions are created, served by the n substations.

If after p iterations of the power Voronoi, there is still any substation whose allocated load is over its capacity, we apply a frontier-correcting algorithm, as explained below. For each iteration of the power Voronoi, the weights of each region are changed, increasing or decreasing, according to the necessity to decrease or increase the size of the region, so as to reach the equilibrium and stability of the system.

5.4 Frontier-correcting algorithm

Using the *n* regions generated by the power Voronoi, a frontier-correcting algorithm (FCA) is applied in the following way: all columns of the mesh are swept searching for vertically neighboring regions such that the capacity of one of these regions is overflown and the other isn't. If a square is in the frontier and belongs to a region with exceeded capacity, it is transferred (that is, the load it represents) to the neighbor region. The application of the FCA for columns is the first iteration of the algorithm. A new gravity center is computed for regions whose capacity has been changed. After this, if there still any substation with its capacity exceeded, the FCA is run again but applied to the lines of the mesh. The whole procedure is repeated alternating its application to columns and lines until there is no substation with overflow.

6. EXPERIMENTS AND RESULTS

Three experiments were run to evaluate the performance of the power Voronoi methodology. In all experiments, 42 power substations were located, based on the work of Corrêa (2003) and using data previously described. However, some of these substations already exist at the time that work was done. Therefore, some of them (22) were considered fixed (those already existing), while others were located by his methodology. For all experiments in this work, we used a fine-grain mesh of squares with 100 X 100 meters. All experiments were run in a PC-clone with Pentium III processor at 1 GHz and 256 Mbytes of RAM.

For all figures in this section, the point within each polygon represents the exact place where a substation is located, together with its identification number. In the companion spreadsheet, variables have the following meaning: *CapSub* = capacity of the substation (in MVA); *CapUt* = amount of the capacity actually used; *PercU* = percent of the capacity actually used; *Área* = area covered by the substation (in km^2). The identification of each region is obtained by the sum of the numbers identifying the column and the line.

6.1 First experiment

In this experiment we consider all 42 substations from the work of Correa (2003): 22 already located and 20 to be located. The existence of fixed substations can make some of them to serve regions to which they not belong.

Figure 4 shows the results obtained after the application of the power Voronoi. A total of 10 iterations were run in 1'34''. Table 1 presents the data corresponding to the distribution of loads to substations.

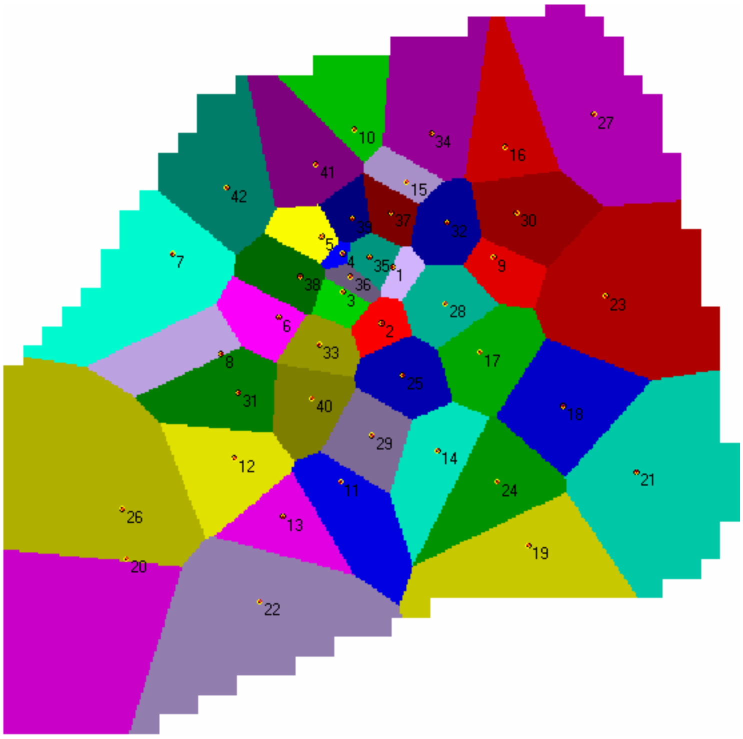

Figure 4 – Power Voronoi diagram for 42 points (22 of them are fixed).

After applying the power Voronoi, the capacity of some substations overflew, and we applied the FCA (section 5.4) aiming at correcting the distribution of loads. After 226 iterations, FCA achieved equilibrium in 56 seconds. The resulting electric moment was 4528043 km*kVA. Figure 5 presents the final distribution of loads to the substations.

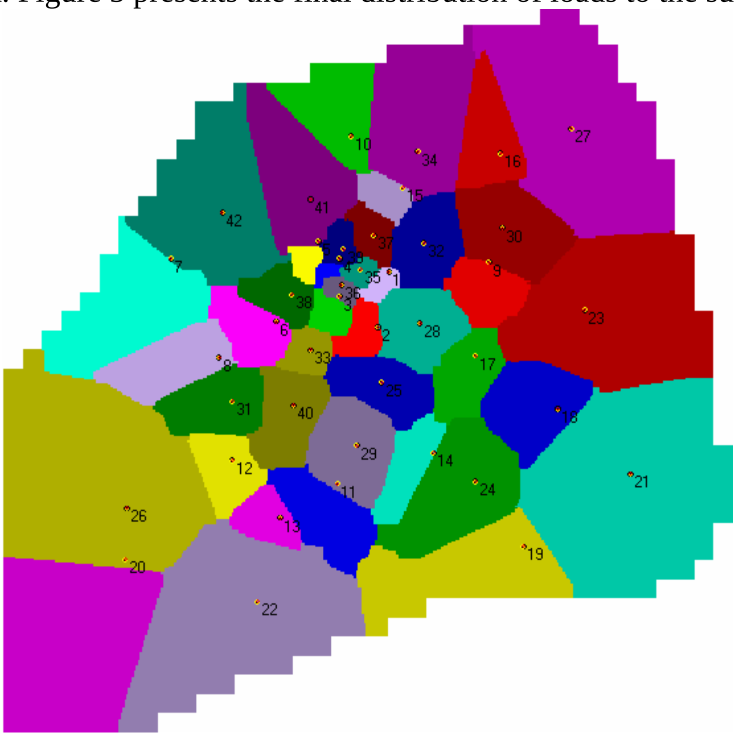

Figure 5 – Result of the application of the FCA to the frontiers
defined by the power Voronoi of Fig. 4.

Table 1 – Distribution of loads for each substation for the first experiment.

+	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área
0	44000	38015	0,86	2,46	46000	45811	1	5,46	80000	80378	1	3,93	40000	36546	0,91	1,08	50000	49793	1	2,73
5	48000	46150	0,96	11,44	40000	40047	1	36,75	40000	37831	0,95	17,56	48000	48168	1	11,07	40000	39887	1	18,89
10	48000	47964	1	17,36	60000	58901	0,98	11,61	60000	59604	0,99	7,78	48000	46340	0,97	10,52	40000	39007	0,98	5,1
15	48000	46190	0,96	15,88	50000	48113	0,96	12,9	48000	47869	1	20,46	40000	39419	0,99	37,31	48000	48067	1	63,55
20	60000	59655	0,99	94,93	40000	38695	0,97	79,21	84000	84409	1	67,78	84000	83807	1	31,65	84000	83427	0,99	14,39
25	84000	83219	0,99	92,99	84000	75916	0,9	77,92	84000	84025	1	16,02	84000	83196	0,99	19,59	84000	82169	0,98	22,51
30	84000	82818	0,99	15,82	84000	83793	1	14,85	84000	80519	0,96	5,41	84000	82115	0,98	36,61	84000	77687	0,92	2,03
35	84000	82892	0,99	2,14	84000	83936	1	5,55	84000	82214	0,98	8,45	84000	82143	0,98	3,69	84000	82763	0,99	15,92
40	84000	81295	0,97	29,52	84000	82338	0,98	48,18												

6.2 Second experiment

This experiment considered a hypothetical situation where all substations should be located anywhere in the mesh, ignoring those already existent or those located by Corrêa (2003). Figure 6 shows the results of 10 iterations of the power Voronoi, taking 1'37''. In this case there is no substation outside its servicing region. Table 2 presents the data corresponding to the distribution of loads to substations.

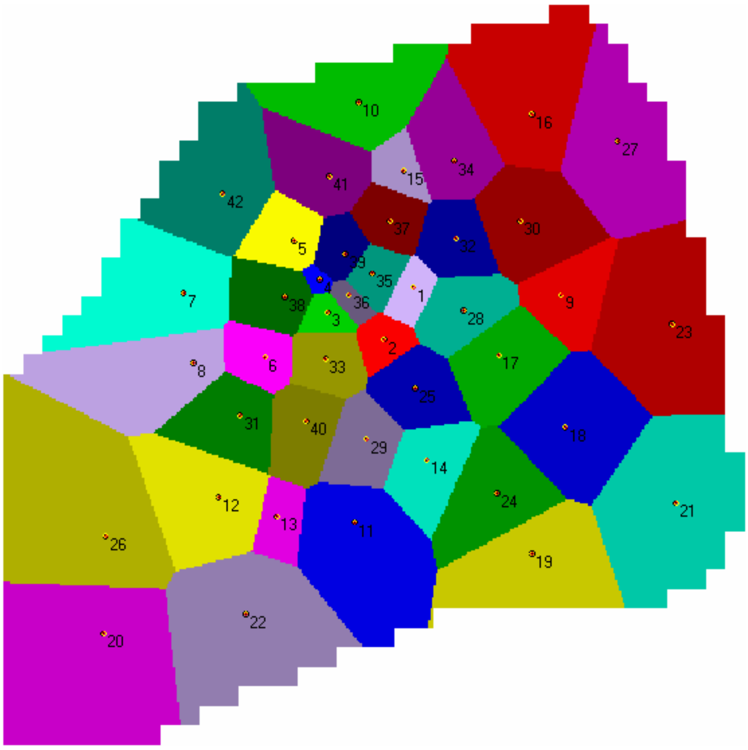

Figure 6 – Power Voronoi diagram for 42 substations, all located by the algorithm.

In the same way as in the first experiment, we run the FCA to further optimize the distribution of loads to the substations. This algorithm run for 183 iterations in 1'28''. The resulting electric moment was 4053456 km*kVA, and Fig. 7 presents the final distribution of loads and the location of substations.

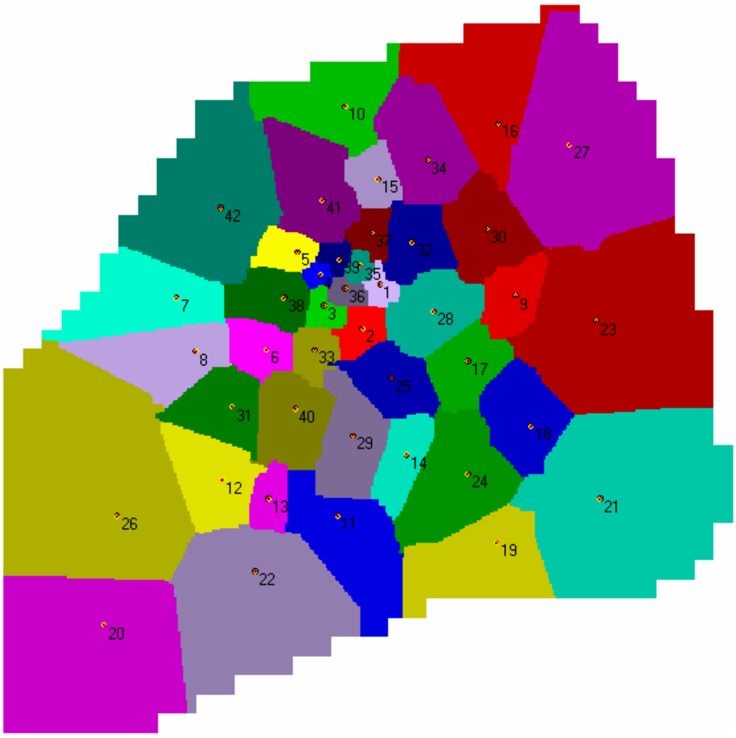

Figure 7 – Result of the application of the FCA to the frontiers defined by the power Voronoi of Fig. 6.

Table 2 – Distribution of loads for each substation for the second experiment.

+	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área
0	44000	36980	0,84	2,66	46000	45386	0,99	5,42	80000	79919	1	3,37	40000	40069	1	1,57	50000	49674	0,99	5,94
5	48000	48105	1	8,53	40000	39033	0,98	26,77	40000	39333	0,98	21,72	48000	46246	0,96	10,41	40000	39905	1	23,95
10	48000	47094	0,98	29,16	60000	59774	1	18,81	60000	57788	0,96	6,16	48000	47056	0,98	10,57	40000	39650	0,99	7,39
15	48000	47195	0,98	35,45	50000	49740	0,99	13,26	48000	47692	0,99	18,41	40000	39669	0,99	29,17	48000	48188	1	73,6
20	60000	56797	0,95	87,53	40000	39527	0,99	62,34	84000	81631	0,97	79,18	84000	81870	0,97	28,62	84000	79966	0,95	13,66
25	84000	83908	1	98,15	84000	80657	0,96	72,57	84000	83891	1	16,51	84000	81719	0,97	17,99	84000	83681	1	18,65
30	84000	82862	0,99	15,15	84000	83179	0,99	12,04	84000	83084	0,99	5,9	84000	80779	0,96	23,5	84000	82187	0,98	2,06
35	84000	83472	0,99	2,66	84000	78955	0,94	4,84	84000	83725	1	11,69	84000	77476	0,92	2,46	84000	82166	0,98	16,65
40	84000	83860	1	22,01	84000	83243	0,99	52,52												

6.3 Third experiment

In this experiment we used the Teitz and Bart algorithm to locate substations, considering that all substations were not located yet. After, we applied the power Voronoi to optimize the distribution of loads, as before. Figure 8 presents the results for 8 iterations of the power Voronoi in 1’30’’. Table 3 presents the data corresponding to the distribution of loads to substations.

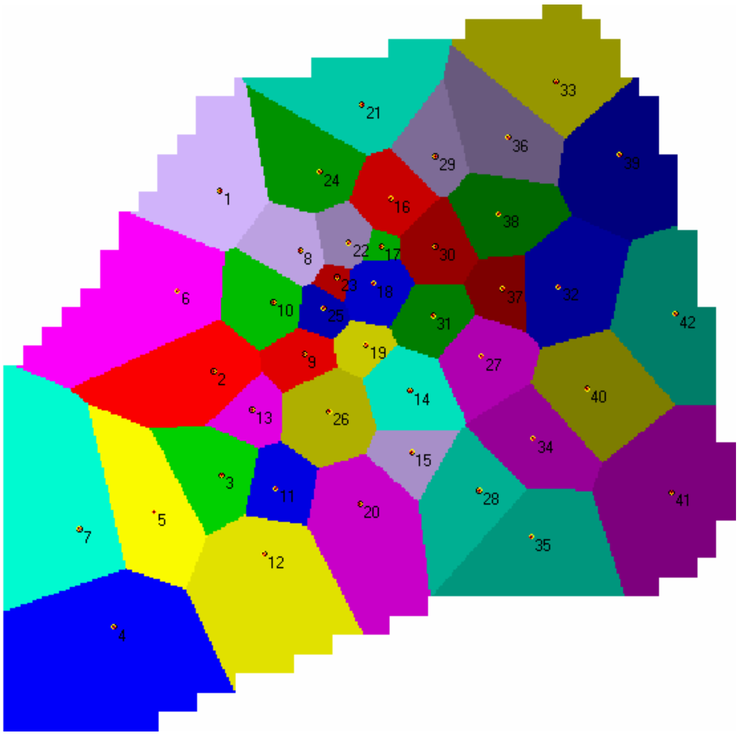

Figure 8 – Power Voronoi diagram for 42 substations.
All of them were located by the Teitz and Bart algorithm.

As before, we applied the FCA to optimize load distribution. This algorithm run for 3’06’’ and the resulting electric moment was 4333640 km*kVA. Figure 9 shows the final distribution of loads and regions to substations.

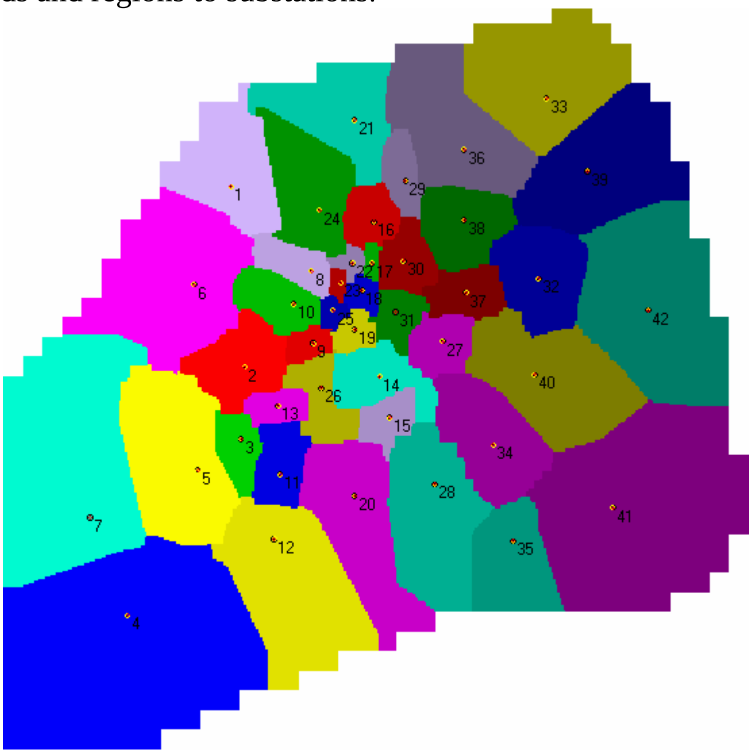

Figure 9 – Result of the application of the FCA to the frontiers
defined by the power Voronoi of Fig. 8.

Table 3 – Distribution of loads for each substation for the third experiment.

+	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	5
	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área	CapSub	CapUt	PercU	Área
0	44000	43179	0,98	26,67	84000	82611	0,98	16,98	46000	44397	0,97	7,55	84000	82554	0,98	115,1!	80000	79674	1	40,89
5	84000	84099	1	51,64	40000	40002	1	75,31	84000	83366	0,99	7,7	50000	49381	0,99	3,47	84000	83277	0,99	10,56
10	48000	45238	0,94	10,16	84000	84250	1	47,75	40000	38682	0,97	4,94	84000	84143	1	13,35	40000	38167	0,95	6,37
15	84000	81045	0,96	7,61	48000	47572	0,99	1,51	84000	84312	1	2,74	40000	37425	0,94	3,78	84000	84119	1	34,82
20	48000	47811	1	24,64	84000	69555	0,83	1,97	60000	55099	0,92	1,24	84000	83353	0,99	23,13	60000	56034	0,93	2,05
25	84000	82804	0,99	9,82	48000	45535	0,95	8,82	84000	82829	0,99	35,71	40000	39287	0,98	6,72	84000	83420	0,99	8,2
30	48000	46751	0,97	6,34	84000	83754	1	20,71	50000	49184	0,98	42,83	84000	83775	1	25,33	48000	47210	0,98	21,11
35	84000	84188	1	36,27	40000	39430	0,99	9,11	84000	79685	0,95	16,35	48000	48214	1	41,52	84000	83593	1	38,58
40	60000	58468	0,97	90,49	40000	39664	0,99	59,07												

7. CONCLUSIONS

The results obtained by our approach were 11.79%, 0.07% and 6.98% higher than the best result found by Corrêa (2003), respectively for the first, second and third experiment. This result is regarding the minimization of the electric moment. Corrêa obtained his results using p-medians for locating substations and the Ford & Fulkerson algorithm for allocating loads. Our approach used mainly the power Voronoi for locating substations together with a frontier-correcting algorithm. The main difference between our results and that of Corrêa is regarding the processing time. The processing time of our algorithm is less than 2 minutes, while Corrêa's algorithm took almost 46 hours. Both systems run in similar computers.

Therefore, our proposed approach is very interesting not only considering the quality of the solutions, but mainly from the computational cost point of view. Results are very promising and encourage further research, especially regarding the use of the Teitz and Bart algorithm for the preliminary location of substations. The methodology allows a significant reduction of workload for engineers in charge of planning the electric power distribution network. Finally, we believe that the methodology proposed can be useful for any other facility location problems.

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